

## DECOHERENCE AND LONG-LIVED SCHRÖDINGER CATS IN BEC\*

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We consider quantum superposition states in Bose–Einstein condensates. A decoherence rate for the Schrödinger cat is calculated and shown to be a significant threat to this macroscopic quantum superposition of BEC’s. An experimental scenario is outlined where the decoherence rate due to the thermal cloud is dramatically reduced thanks to trap engineering and “symmetrization” of the environment. We show that under the proposed scenario the Schrödinger cat belongs to an approximate decoherence-free pointer subspace.

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### 1. Introduction

Microscopic quantum superpositions are an everyday physicist’s experience. Macroscopic quantum superpositions, despite nearly a century of experimentation with quantum mechanics, are still encountered only very rarely. Fast decoherence of macroscopically distinct states is to be blamed [1]. In spite of that, recent years were a witness to an interesting quantum optics experiment [2] on decoherence of a few photon superpositions. Moreover, matter-wave interference in fullerene C<sub>60</sub> has been observed [3]. Another experiment has succeeded in “engineering” the environment in the context of trapped ions [4]. Recently the first detection of a macroscopic Schrödinger cat state in an rf-SQUID was reported [5]. All these successes tempt one to

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push similar investigations of basic quantum mechanics into the rapidly progressing field of Bose–Einstein condensation (BEC) of alkali metal atomic vapors [6]. The condensates can contain up to  $10^7$  atoms in the same quantum state. What is more, it is possible to prepare condensates in two different internal states of the atoms. Some of these pairs of internal levels are immiscible, and their condensates tend to phase separate into distinct domains with definite internal states [7]. The immiscibility seems to be a prerequisite to prepare a quantum superposition in which all atoms are in one or the other internal state,  $|\psi\rangle = (|N, 0\rangle + |0, N\rangle)/\sqrt{2}$ , where  $N$  is the total number of condensed atoms. There are at least two theoretical proposals how to prepare a macroscopic quantum superposition in this framework [8, 9]. Other proposals involve non-unitary evolution of the BEC towards the Schrödinger cat state by means of continuous quantum measurements [10, 11]. Neither of these proposals addresses the crucial question of decoherence.

This paper is a simplified and more pedagogical version of our previous work [12]. All technical details of the calculations, in particular the computation of the decoherence rate for the BEC superposition state, can be found in that reference.

## 2. The condensate

Let us first make a short summary of some of the ideas for creating quantum superposition states of Bose–Einstein condensates. The methods that have been thus far proposed start with two weakly interacting dilute Bose condensates of atoms in different internal states. Let us call these internal states  $A$  and  $B$ . Atoms interact through  $s$ -wave collisions, characterized by a single parameter  $a$  which is known as the scattering length. We shall consider the case of repulsive interaction  $a > 0$ ; it is known that for the opposite case the condensate is unstable above a critical total number of particles. It is assumed that the inter-scattering lengths for collisions  $A$ – $A$  and  $B$ – $B$  are the same, *i.e.*  $a_{AA} = a_{BB}$ , which is in general different from the intra-scattering length  $a_{AB}$ . When the self-energy of atom–atom interactions of the BEC is much less than the mode energy spacing, then one can treat the condensate in the two-mode approximation. Moreover, the condensate is shined with a appropriately chosen laser that introduces a Josephson-like coupling of strength  $\lambda$  that interchanges internal atomic states in a coherent manner. The condensate two-mode Hamiltonian is

$$H_C = \varepsilon_g(a^\dagger a + b^\dagger b) - \lambda(a^\dagger b + b^\dagger a) + \frac{u_c}{2}(a^\dagger a^\dagger a a + b^\dagger b^\dagger b b) + v_c(a^\dagger b^\dagger a b). \quad (1)$$

Here  $\varepsilon_g$  is the energy of the lowest single-particle state in the trap (assumed to be the same for  $A$  and  $B$ ),  $u_c = 4\pi\hbar^2 a_{AA}/m$  and  $v_c = 4\pi\hbar^2 a_{AB}/m$ .

This Hamiltonian was studied in great detail in [8, 9, 13]. The precise quantum state of the condensate for all values of  $u_c$  and  $v_c$  was studied by means of a Wigner-like distribution on the two-mode Bloch sphere. It was found that when the repulsion between atoms in different states is bigger than the repulsion between atoms in the same state, that is when  $v_c > u_c$ , then the condensate tends to phase separate in space into distinct domains with definite internal states. This condition is known as the immiscibility condition. More importantly, the ground state of the system is a quantum superposition state. When the ‘‘purity’’ factor defined as  $\varepsilon \equiv (\lambda/(v_c - u_c)N)^N$  is much less than one, then the lowest energy subspace contains two macroscopic superpositions

$$\begin{aligned} |+\rangle &= \frac{1}{\sqrt{2} N!} [(a^\dagger)^N + (b^\dagger)^N] |0, 0\rangle \equiv \frac{1}{\sqrt{2}} (|N, 0\rangle + |0, N\rangle), \\ |-\rangle &= \frac{1}{\sqrt{2} N!} [(a^\dagger)^N - (b^\dagger)^N] |0, 0\rangle \equiv \frac{1}{\sqrt{2}} (|N, 0\rangle - |0, N\rangle). \end{aligned} \quad (2)$$

The lower  $|+\rangle$  and the higher  $|-\rangle$  states are separated by a small energy gap of  $E_- - E_+ = N(u_c - v_c)\varepsilon \ln \varepsilon$ . In the extreme case of  $\lambda = 0$  (no laser applied) the two states are completely degenerate. If we just have  $\varepsilon < 1$ , the  $|\pm\rangle$  states in fact contain an admixture of intermediate states  $|N-1, 1\rangle, \dots, |1, N-1\rangle$  such that their overlap is  $\langle + | - \rangle = \varepsilon$ . For  $\varepsilon \gg 1$  they shrink to

$$(a^\dagger \pm b^\dagger)^N |0, 0\rangle. \quad (3)$$

which does not correspond to a macroscopic superposition. From now on we assume the pure case,  $\varepsilon \ll 1$ .

The proposal of Gordon and Savage [9] for obtaining these superposition states is similar to a typical experiment in quantum optics with non-linear systems. In the BEC case, the non-linearity is provided by the collisional interactions and the Josephson coupling. The idea is to prepare an initial state with all the atoms in the same internal state and then turn on the Josephson coupling for some amount of time adequately chosen. After the Josephson coupling is turned off, the quantum state of the system has been modified, and a Schrödinger cat state has been formed. Other proposals for obtaining basically the same kind of superposition use a continuous quantum measurement process [11].

### 3. The thermal cloud

As we have already mentioned, neither of the proposals for generating quantum superpositions in BEC addresses the crucial question of decoherence. In fact, the condensate is an open quantum system which is in contact with an environment of non-condensed thermal particles. This interaction may be responsible for the loss of coherence between the components of the quantum superposition states of the previous section. If the decoherence time were very small, then the existence of these states in BEC would be merely of academic interest, since the expectations of observing them in the lab would vanish. Therefore it is important to understand how the thermal cloud affects the longevity of the BEC cats.

The Hamiltonian for the dilute environment formed by non-condensed particles is

$$H_E = \sum_s \left[ \varepsilon_s (a_s^\dagger a_s + b_s^\dagger b_s) - \lambda (a_s^\dagger b_s + b_s^\dagger a_s) \right], \quad (4)$$

where  $\varepsilon_s$  is the single particle energy of level  $s$ . After the transformation

$$S_s = \frac{a_s + b_s}{\sqrt{2}}, \quad O_s = \frac{a_s - b_s}{\sqrt{2}} \quad (5)$$

$H_E$  adopts a diagonal form

$$H_E = \sum_s \left[ (\varepsilon_s - \lambda) S_s^\dagger S_s + (\varepsilon_s + \lambda) O_s^\dagger O_s \right]. \quad (6)$$

In figure 1 we make a schematic plot of the energy levels of the non-condensed system before ( $\lambda = 0$ ) and after ( $\lambda \neq 0$ ) applying the Josephson coupling. For simplicity we consider an isotropic harmonic trap with a dip in its center, where the condensate particles will be localized (we shall return to this point later). We see that when the laser coupling is applied, the states  $S_s$  (which are symmetric under the exchange  $a \leftrightarrow b$ ) and the states  $O_s$  (which are antisymmetric under the same exchange) feel the same, but shifted, trapping potentials. The two ladders of energy eigenstates of the environment are separated by a gap equal to  $2\lambda$ . Each of the levels is occupied according to the thermal Bose distribution. When the gap is much bigger than the typical thermal energy  $2\lambda \gg k_B T$ , then the antisymmetric states are unoccupied. We will see in the next section that this is a prerequisite for reducing decoherence.

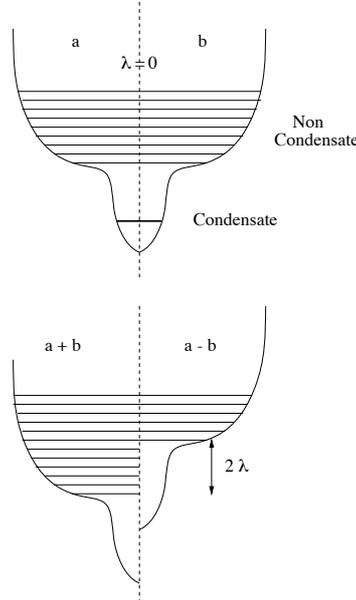


Fig. 1. Environmental single-particle energy levels before and after turning on the Josephson-like coupling.

#### 4. Condensate–non-condensate interactions

Condensed atoms interact with non-condensed atoms via two-body collisions. The exact interaction Hamiltonian  $V$  can be easily obtained from the total (Gross–Pitaevskii like) Hamiltonian

$$\begin{aligned}
 H = \int d^3x & \left[ v(\phi_A^\dagger \phi_A)(\phi_B^\dagger \phi_B) \right. \\
 & \left. + \left\{ \frac{u}{2} \phi_A^\dagger \phi_A^\dagger \phi_A \phi_A + \nabla \phi_A^\dagger \nabla \phi_A + U(r) \phi_A^\dagger \phi_A - \lambda \phi_A^\dagger \phi_B \right\} + \{A \leftrightarrow B\} \right], \quad (7)
 \end{aligned}$$

after splitting the quantum fields  $\phi_A$  and  $\phi_B$  into condensate and bath parts, *i.e.*

$$\begin{aligned}
 \phi_A(\vec{x}) &= a g(\vec{x}) + \sum_s a_s u_s(\vec{x}), \\
 \phi_B(\vec{x}) &= b g(\vec{x}) + \sum_s b_s u_s(\vec{x}), \quad (8)
 \end{aligned}$$

where  $g(\vec{x})$  is the wave function of the condensate localized in the dip. After introducing the symmetric and antisymmetric operators  $S_s$  and  $O_s$  for the thermal bath, the interaction Hamiltonian can be split into two terms,

$V = V_S + V_O$ , which are respectively symmetric and antisymmetric under the interchange  $a \leftrightarrow b$  of system operators. For the sake of conciseness, we shall refrain from writing down the whole expressions for these two interaction Hamiltonians. Suffice it to say that each of them has two possible energy-conserving interaction vertices, both proportional to the interaction couplings  $u$  or  $v$ :

- There are inelastic two-body collisions that do not conserve the number of condensed particles (condensate feeding and depletion processes). There are in turn two possible diagrams for these processes, and they are shown in figure 2. The first one is proportional to the occupation number of initial non-condensed states; since each occupation number is proportional to fugacity  $z \equiv e^{\beta\mu}$  (where  $\beta$  is the inverse temperature of the thermal bath and  $\mu$  its chemical potential), then the diagram is  $O(z^2)$ . The second one is proportional to the occupation number of the initial non-condensed state (that is, proportional to  $z$ ), and energy conservation brings about another fugacity factor, so the final result is that this diagram is also  $O(z^2)$  [14, 15].
- There are elastic two-body collisions that conserve the number of condensed particles, and are shown in figure 3. These diagrams are proportional to fugacity,  $O(z)$  [14, 15].

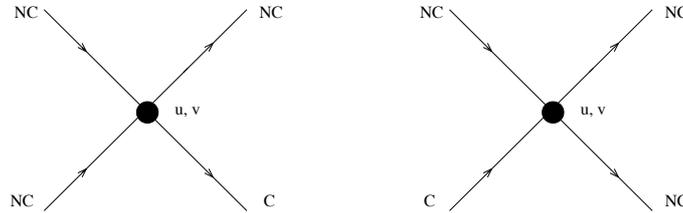


Fig. 2. Inelastic two-body collisions between condensate and non-condensate particles.

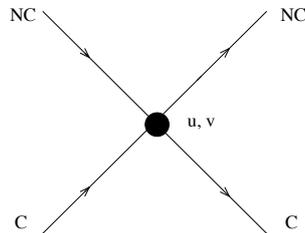


Fig. 3. Elastic two-body collisions between condensate and non-condensate particles.

When fugacity is small (which occurs when the gap between the condensate and non-condensate single-particle levels is bigger than the thermal energy  $k_B T$ ), then the elastic processes dominate over the inelastic ones. The elastic collisions are also the most relevant ones in the computation of the decoherence rate, since we shall see that their contribution scales as  $N^2$  (with  $N$  the total number of particles in the condensate), whereas the inelastic contribution scales as  $N$ . For these reasons, from now on we shall concentrate only on the elastic processes.

### 5. Decoherence-free pointer subspace in BEC

As we have mentioned in Section 3, when  $2\lambda \gg k_B T$  the antisymmetric environmental states are nearly empty. Only the symmetric states are occupied. Since these latter states cannot distinguish between  $A$  and  $B$ , all collisions that involve symmetric non-condensed states will not destroy the quantum phase coherence between the Schrödinger cat's components. We refer to this condition as the perfect symmetrization limit. In this case, the states  $|\pm\rangle \equiv (|N, 0\rangle \pm |0, N\rangle)/\sqrt{2}$  span a decoherence-free pointer subspace (also known in the literature as decoherence-free subspace or, for short, DFS) of the Hilbert space, since they have degenerate eigenvalues of the interaction Hamiltonian  $V$  [16–18]. Any state of that subspace can be written as  $\alpha|N, 0\rangle + \beta|0, N\rangle$ , with  $\alpha$  and  $\beta$  complex numbers. If  $\mathcal{P}_{[\alpha|N,0\rangle + \beta|0,N\rangle]}$  denotes a projector onto that subspace, then

$$[V, \mathcal{P}_{[\alpha|N,0\rangle + \beta|0,N\rangle]}] = 0, \quad (9)$$

which means that any quantum superposition  $\alpha|N, 0\rangle + \beta|0, N\rangle$  in the subspace is an eigenstate of the interaction Hamiltonian (a perfect pointer state), and as such will retain its phase coherence and last forever. The interaction Hamiltonian between the condensate and the thermal cloud is a sum of products of condensate operators and environmental operators. Only terms with symmetric environmental operators are relevant because the antisymmetric states are empty. The total Hamiltonian is symmetric with respect to  $A \leftrightarrow B$  so, to preserve this symmetry, the relevant terms with symmetric environmental operators also contain symmetric condensate operators. The argument simplifies a lot for small fugacity where there is only one leading term with the  $N_A + N_B$  condensate operator. The states  $|\pm\rangle$  are its eigenstates with the same eigenvalue  $N$ . They are also (almost) degenerate eigenstates of the condensate Hamiltonian build out of  $N_{A,B}$ . The coherent transitions  $A \leftrightarrow B$  break this degeneracy of  $|\pm\rangle$  but the difference of their eigenfrequencies is negligible as compared to the usual condensate lifetime of  $\sim 10$  s. In the next-to-leading order in fugacity there are symmetric interaction terms which change the number of condensed atoms. These

terms drive the  $|N, 0\rangle$  and  $|0, N\rangle$  states into slightly “squeezed-like” states  $|S, 0\rangle$  and  $|0, S\rangle$  respectively [22]. There are also terms which exchange  $A$  with  $B$ . They give each state a small admixture of the opposite component. Superpositions of these are still decoherence-free pointer subspaces — there are no relevant antisymmetric operators to destroy their quantum coherence.

## 6. Decoherence comes into the scene

When the antisymmetric environmental states begin to be occupied, then the commutation relation (9) is only approximate and states within the subspace will decohere. Hence we must face the problem of calculating decoherence rates due to interactions with the environment.

We shall first consider the case of  $\lambda = 0$ , which corresponds to no Josephson-coupling being applied, and for which the two states  $|\pm\rangle$  are exactly degenerate with respect to the condensate Hamiltonian. In this case there is no difference between symmetric and antisymmetric environmental states. After some long but straightforward calculation which involves computing the master equation for the reduced density matrix of the condensed particles (see [12] for details), we find a lower bound for the decoherence rate

$$t_{\text{dec}}^{-1} > 16\pi^3 \left( 4\pi a^2 \frac{N_E}{V} v_T \right) N^2, \quad (10)$$

where  $N$  is a number of condensed atoms,  $v_T = \sqrt{2k_B T/m}$  is a thermal velocity in the noncondensed thermal cloud,  $a$  is a scattering length,  $V$  is a volume of the trap, and  $N_E$  is a number of atoms in the thermal cloud,

$$N_E \approx e^{\frac{\mu}{k_B T}} \left( \frac{k_B T}{\hbar\omega} \right)^3. \quad (11)$$

This is a lower estimate since we have only considered terms to leading order in fugacity  $O(z)$  and to leading order in condensate size  $O(N^2)$ . Next-to-leading order terms are  $O(z^2)$  and  $O(N)$ , in agreement with Refs. [14,15], so they were neglected here. Even without going into details of our derivation it is easy to understand where a formula like Eq. (10) comes from.  $N^2$  is the main factor which makes the decoherence rate large. It comes from the master equation of the Bloch–Lindblad form  $\dot{\rho} \sim -[N_A - N_B, [N_A - N_B, \rho]]$ , with  $A$  and  $B$  the two internal states of the atoms.  $N^2$  is the distance squared between macroscopically different components of the superposition  $(|N, 0\rangle + |0, N\rangle)/\sqrt{2}$  — the common wisdom reason why macroscopic objects are classical [1]. The factor in brackets in Eq. (10) is a scattering rate of a condensate atom on noncondensate atoms — the very process by which the thermal cloud environment learns the quantum state of the system.

Let us estimate the decoherence rate for a set of typical parameters:  $T = 1 \mu\text{K}$ ,  $\omega = 50 \text{ Hz}$ , and  $a = 3 \dots 5 \text{ nm}$ . The thermal velocity is  $v_T \approx 10^{-2} \text{ m/s}$ . The volume of the trap can be approximated by  $V = 4\pi a_{\text{return}}^3/3$ , where  $a_{\text{return}} = \sqrt{2k_B T/m\omega^2}$  is a return point in a harmonic trap at the energy of  $k_B T$ . We estimate the decoherence time as  $t_{\text{dec}} \approx 10^5 \text{ sec}/(N_E N^2)$ . For  $N_E = 10^0 \dots 10^4$  and  $N = 10 \dots 10^7$  it can range from 1000s down to  $10^{-13} \text{ s}$ . For  $N = 10$  our (over-)estimate for  $t_{\text{dec}}$  is large. However, already for  $N = 1000$  and  $N_E = 10$  (which are still within the limits of validity of the two-mode approximation)  $t_{\text{dec}}$  shrinks down to milliseconds. Given that our  $t_{\text{dec}}$  is an upper estimate and that big condensates are more interesting as Schrödinger cats, it is clear that for the sake of cat's longevity, one must go beyond the standard harmonic trap setting.

## 7. Trap engineering

From Eqs. (10), (11) it is obvious that the decoherence rate depends a lot on temperature and on chemical potential. The two factors strongly influence both  $N_E$  and  $v_T$ . Both can be improved by the following scenario, which is a combination of present day experimental techniques. In the experiment of Ref. [19] a narrow optical dip was superposed at the bottom of a wide magnetic trap. The parameters of the dip were tuned so that it had just one bound state. The gap between this single condensate mode and the first excited state was  $1.5 \mu\text{K}$ , which at  $T = 1 \mu\text{K}$  gives a fugacity of  $z = \exp(-1.5)$ . We need the gap so that we can use the single mode approximation. At low temperatures, the gap results in a small fugacity, which is convenient for calculations. We propose to prepare a condensate inside a similar combination of a wide magnetic and a narrow optical trap (or more generally: a wide well plus a narrow dip with a single bound mode) and then to open the magnetic trap and let the noncondensed atoms disperse. The aim is to get rid of the thermal cloud as much as possible. A similar technique was used in the experiment of Ref. [20] (see figure 4).

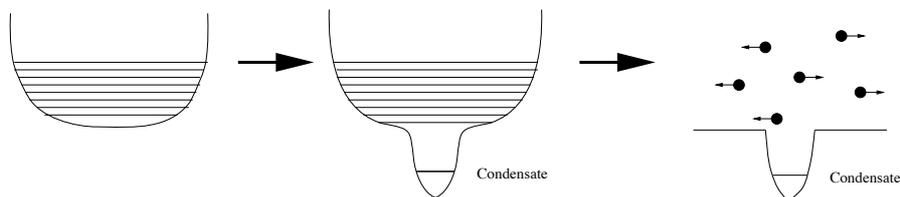


Fig. 4. First step in engineering the trap: an optical dip is superimposed on a big magnetic trap, which is later opened so that most of the thermal atoms disperse away.

Let us estimate the ultimate limit for the efficiency of this technique. At the typical initial temperature of  $1\mu\text{K}$  the thermal velocity of atoms is  $10^{-2}\text{ m/s}$ . An atom with this velocity can cross a  $1\mu\text{m}$  dip in  $10^{-4}\text{ s}$ . If we wait for, say,  $1\text{ s}$  after opening the wide trap, then all atoms with velocities above  $10^{-6}\text{ m/s}$  will disperse away from the dip. A thermal velocity of  $10^{-6}\text{ m/s}$  corresponds to the temperature of  $10^{-8}\mu\text{K}$ . As the factor  $N_{\text{E}}v_T \sim T^{7/2}$  in Eq. (10), then already  $1\text{ s}$  after opening the wide trap the decoherence rate due to non-condensed atoms is reduced by a factor of  $10^{-28}$ !

It is not realistic to expect such a ‘‘cosmological’’ reduction factor. The ‘‘dip’’ which is left after the wide harmonic trap is gone could be, for example, a superposition of an ideal dip plus a wide shallow well (which was a negligible perturbation in presence of the wide harmonic trap). The well would have a band of width  $\Delta E$  of bound states which would not disperse but preserve their occupation numbers from before the opening of the wide trap. They would stay in contact with the condensate and continue to ‘‘monitor’’ its quantum state. Even if such a truncated environment happens to be already relatively harmless, there are means to do better than that.

Further reduction of the decoherence rate can be achieved by ‘‘symmetrization’’ of the environmental states. To this end we propose to turn on the Josephson-like coupling  $\lambda$  in order to induce coherent transitions between states  $A$  and  $B$ . This has already been achieved experimentally [21]; in their case  $\lambda \approx 1\text{ kHz}$ . When  $\Delta E \ll 2\lambda$  we are in the perfect symmetrization limit, and hence no decoherence takes place. Indeed, the symmetric and antisymmetric  $\Delta E$ -bands of states can be visualized as two ladders shifted with respect to each other by  $2\lambda$ . In other words, the two sets of states feel the same, but shifted, trapping potentials. When  $\Delta E \ll 2\lambda$ , then the antisymmetric  $O_s$ 's are nearly empty since they can evaporate into symmetric states and then leave the trap. The symmetric  $S_s$ 's cannot distinguish between  $A$  and  $B$  so they do not destroy the quantum coherence between the Schrödinger cat's components. After symmetrization,  $N_{\text{E}}$  in Eq. (10) has to be replaced by the final number of atoms in the antisymmetric states only;

$$N_{\text{E}}^O \approx \begin{cases} n_\lambda \exp((\mu - \lambda)/k_{\text{B}}T), & \text{for } 2\lambda < \Delta E, \\ 0, & \text{for } 2\lambda > \Delta E. \end{cases} \quad (12)$$

Here  $T$  is the temperature before opening the wide trap.  $n_\lambda$  is the number of antisymmetric bound states which remain within the  $\Delta E$ -band of symmetric states. Atoms in these antisymmetric bound states cannot disperse away. For  $2\lambda > \Delta E$  this number  $n_\lambda$  is zero and there is no decoherence from the thermal cloud.

In figure 5 we make a schematic graph of the steps we propose to engineer the trap in order to reduce decoherence due to the thermal cloud.

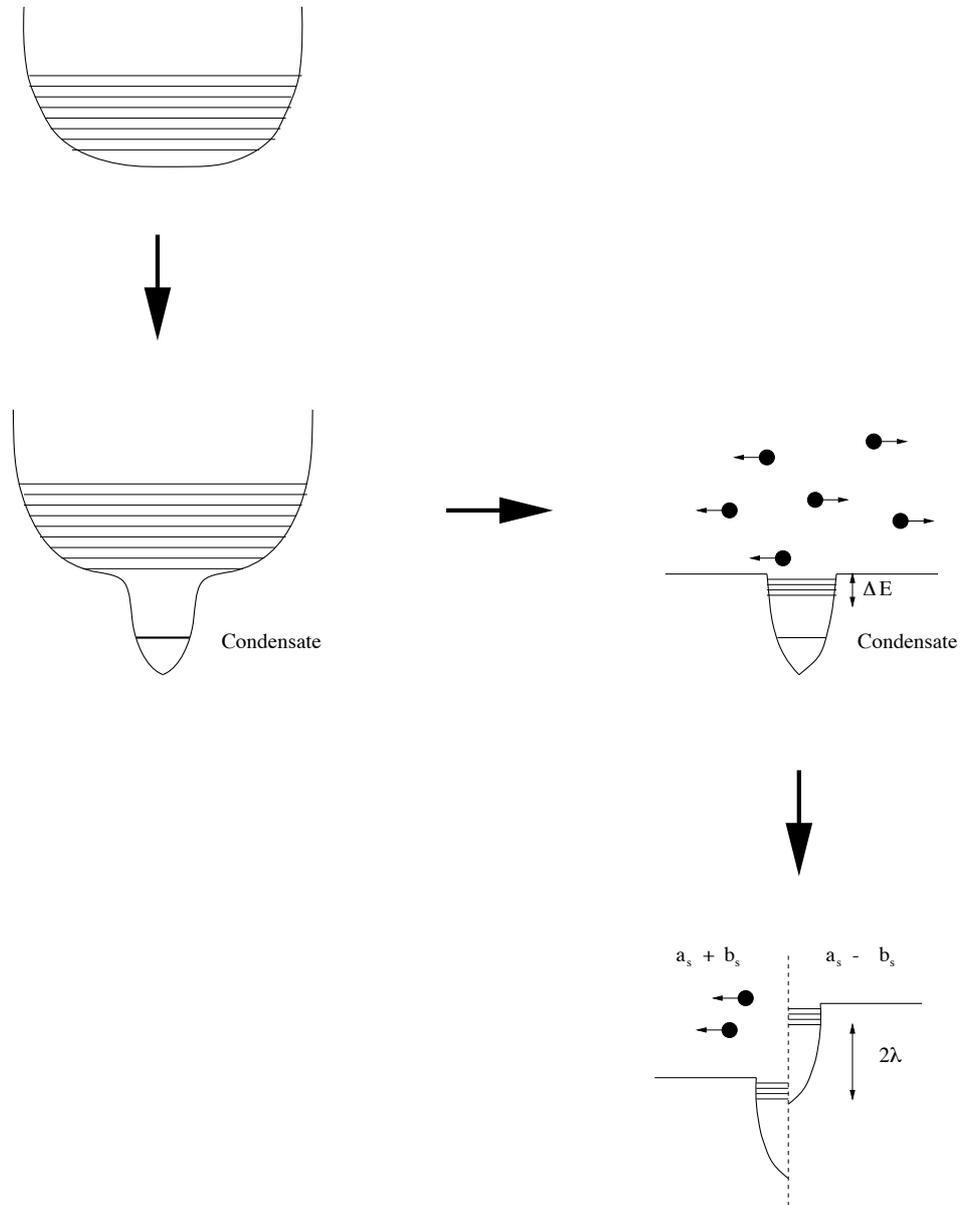


Fig. 5. Second step in engineering the trap: After the magnetic trap is opened, there is still a band  $\Delta E$  of thermal states in the “mouth” of the dip. Turning on the Josephson coupling  $\lambda$  shifts the energy levels of symmetric and antisymmetric thermal states, and the latter become empty.

### 8. Other sources of decoherence

Interaction with the thermal atoms is not the only source of decoherence for the quantum superposition state of the condensate. Among other possible sources we can mention:

- Ambient magnetic fields: the condensed atoms have magnetic moments. If the magnetic moments of  $A$  and  $B$  were different the magnetic field would distinguish between them and would introduce an unknown phase into the quantum superposition, thus rendering its underlying coherent nature undetectable. Fortunately the much used  $|F, m_F\rangle = |2, 1\rangle, |1, -1\rangle$  states of  $^{87}\text{Rb}$  have the same magnetic moments. For them the magnetic field is a “symmetric” environment.
- Different scattering lengths: there is a typical  $\approx 1\%$  difference between the  $A$ - $A$  and  $B$ - $B$  scattering lengths. The condensate Hamiltonian is not perfectly symmetric under  $A \leftrightarrow B$ . Even for a perfectly symmetrized environment, symmetric environmental operators couple to not fully symmetric condensate operators. This means that for the  $1\%$  difference of scattering lengths symmetrization can improve decoherence time by at most two orders of magnitude as compared to the unsymmetrized environment.
- Three-body losses: due to collisions involving three particles, the condensate is not stable but loses atoms via recombination into molecular states. Hence a condensate has a finite lifetime, of the order of  $10 \dots 20$  s. The atoms which escape from the condensate carry information about its quantum state. They destroy its quantum coherence. In the experiment of Ref. [19] the measured loss rate per atom was  $4/s$  for  $N = 10^7$  or around 1 atom per  $10^{-7}$  s. The loss rate scales like  $N^3$  so already for  $N = 10^4$  just one atom is lost per second; decoherence time is 1 s. One possibility for increasing this decoherence time is to increase slightly the dip radius. The loss rate scales like density squared so an increase in the dip width by a factor of 2 reduces the loss rate by a factor of  $2^6 = 64$ .

### 9. Discussion

The aim of this paper was to discuss the “longevity” of Schrödinger cats in BEC’s. We have shown that while in the standard traps decoherence rates are significant enough to prevent long-lived macroscopic superpositions of internal states of the condensate, the strategy of trap engineering and symmetrization of the environment will be able to deal with that issue.

What remains to be considered is how one can generate such macroscopic quantum superposition, and how one can detect it. The issue of generation was already touched upon in Refs. [8, 9]. We have little to add to this. However, in the context of the Gordon and Savage proposal, it is fairly clear that the time needed to generate the cat state would have to be short compared to the decoherence time. If our estimates of Eq. (10) are correct, symmetrization procedure appears necessary for the success of such schemes.

Detection of Schrödinger cat states is perhaps a more challenging subject. In principle, states of the form  $(|N, 0\rangle + |0, N\rangle)/\sqrt{2}$  have a character of GHZ states, and one could envision performing measurements analogous to those suggested in [27] and carried out in [28], where a 4-atom entangled state was studied. However, this sort of parity-check strategy, appropriate for  $N \leq 10$ , is likely to fail when  $N$  is larger, or when (as would be the case for the “quasi-squeezed” states anticipated here [22])  $N$  is not even well defined.

A strong circumstantial evidence can be nevertheless obtained from two measurements. The first one would consist of a preparation of the cat state, and of a measurement of the internal states of the atoms. It is expected that in each instance all (to within the experimental error) would turn up to be in either  $A$  or  $B$  states. However, averaged over many runs, the number of either of these two alternatives would be approximately equal. Decoherence in which the environment also “monitors” the internal state of the atoms in the  $A$  versus  $B$  basis would not influence this prediction. We need to check separately whether the cat state was indeed coherent. To do this, one could evolve the system “backwards”. However, this is not really necessary. For, as Gordon and Savage point out, when, in their scheme, we let the system evolve unitarily for more or less twice the time needed for the generation of the cat state, it will approximately return to the initial configuration. Thus, we can acquire strong evidence of the coherence of the cat provided that this unitary return to the initial configuration can be experimentally confirmed.

These are admittedly rather vague ideas, which serve more as a “proof of principle” rather than as a blueprint for an experiment. Nevertheless, they may, we hope, encourage more concrete investigation of such issues with a specific experiment in mind.

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